Homework 6

MATH 301/601

Due Wednesday, March 20, 2024

Instructions. *Read the appropriate homework guide (Homework Guide for 301 or Homework Guide for 601) to make sure you understand how to successfully complete the assignment. All claims must be sufficiently justified.*

Exercise 1. Complete the following exercises from #1 from Section 4.5 in the course textbook: # 24, 25, 29, ***34**, 36

Exercise 2. Prove that the order of S_n is $n!$.

Exercise 3. (a) Write down the elements in the cyclic subgroups generated by the cycles (1 4 3) and (1 3 5 2 4 6).

(b) Prove the order of a k-cycle is k.

Exercise 4. Complete the following exercises from Section 5.4 in the course textbook:

#1, 2(a,b,c,d), 4, 5 (ignore the first sentence, and just find each of the sets in a, b, and c, and decide whether they are subgroups or not), 17, 23, ***33**

***Exercise 5.** Prove that a k-cycle can be expressed as the product of $k-1$ transpositions. (Hint: You should use induction.)

****Exercise 6.** Prove that any two k-cycles in S_n are conjugate, that is, if $\sigma, \tau \in S_n$ are k-cycles, then there exists $\mu \in S_n$ such that $\mu\sigma\mu^{-1} = \tau$.