Homework 12

MATH 301

Not to be collected, but problems are relevant to Exam 3

Exercise 1. Complete the following exercises from Section 16.7:

1(a)–(g), 7, 13(a)–(c)

Exercise 2. Prove that the function φ: $\mathbf{F}_4 \to \mathbf{F}_4$ given by $\varphi(x) = x^2$ is an isomorphism. (We defined $\mathbf{F}_4$ in class.)

Exercise 3. The goal here is to explore the field of order 9.

(a) Find an irreducible quadratic polynomial p in $\mathbb{Z}_3[x]$.

(b) Then, $\mathbb{F}_9 = \{a + b\beta : a, b \in \mathbb{Z}_3 \text{ and } p(\beta) = 0\}$ is a field of order 9. Find the inverses of $1 + \beta, 2 + \beta$, and $1 + 2\beta$.