Homework 8

MATH 301

Due Wednesday, November 15, 2023

Instructions. *Read the Homework Guide to make sure you understand how to successfully complete the assignment. All claims must be sufficiently justified.*

***Exercise 1.** (a) Find all the subgroups of $\mathbb{Z}_{26}$. Justify your answer.

(b) Let $p \in \mathbb{N}$ be prime. How many subgroups does $\mathbb{Z}_{2p}$ have? Prove it.

Exercise 2. Let $A \in \mathrm{GL}(n, \mathbb{R})$ be a matrix with odd order. Prove that $A \in \mathrm{SL}(n, \mathbb{R})$.

Exercise 3. Let G be a group, and given $g \in G$, define $\lambda_g \colon G \to G$ by $\lambda_g(h) = gh$ for $h \in G$. Prove that: $\psi \colon G \to \mathrm{Sym}(G)$ given by $\psi(g) = \lambda_g$ is injective.

Exercise 4. Give a set of matrices in $\mathrm{GL}(4, \mathbb{R})$ that form a group isomorphic to D_4.

Definition 1. An *automorphism* of a group G is an isomorphism $G \to G$.

***Exercise 5.** Let G be a finite abelian group of order m. Suppose $n \in \mathbb{N}$ is relatively prime to m. Prove that $\varphi \colon G \to G$ given by $\varphi(g) = g^n$ is an automorphism of G. (This says that every element of G has an n^{th}-root.)

Exercise 6. Let G be a group. Prove that the set of automorphisms of G, denoted $\mathrm{Aut}(G)$, is a group with respect to function composition (this group is called *the automorphism group of* G).

Exercise 7. Complete the following exercises from Section 9.4:

8, 12, 38, ***39**, 41

****Exercise 8.** Prove that every automorphism of S_3 is an inner automorphism.