

## Homework 6

Due Wednesday, October 25, 2023

MATH 301

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**Instructions.** Read the [Homework Guide](#) to make sure you understand how to successfully complete the assignment. All claims must be sufficiently justified.

**Exercise 1.** Prove that the order of  $S_n$  is  $n!$ .

**Exercise 2.** (a) Write down the elements in the cyclic subgroups generated by the cycles  $(1\ 4\ 3)$  and  $(1\ 3\ 5\ 2\ 4\ 6)$ .

(b) Prove the order of a  $k$ -cycle is  $k$ .

**Exercise 3.** Complete the following exercises from [Section 5.4](#) in the course textbook:

#1, 2(a,b,c,d), 4, 5 (ignore the first sentence, and just find each of the sets in a, b, and c, and decide whether they are subgroups or not), 7, 8, 17, 18, 23, \*25, \*33

**\*Exercise 4.** From Exercise 1,  $|S_4| = 4! = 24$ . Show that for any divisor  $d$  of 24 there exists a subgroup  $H$  such that  $|H| = d$ .

**\*\*Exercise 5.** Prove that  $A_9$  is generated by two 5-cycles. (This means there are two five cycles, say  $\sigma$  and  $\tau$ , such that every element in  $A_9$  can be expressed as  $w_1 w_2 \cdots w_k$ , where  $w_i \in \{\sigma, \tau\}$ .)

This question came up in discussing the [Gizmo Gear Puzzle](#), see also this [research paper about the puzzle](#) (one of the authors is a current master's student at QC). Understanding Figure 3(b) is where the question comes from, and it might also help in doing the exercise!