

Instructions. *This assignment will not be collected; but, the material will be on Exam 3.*

Exercise 1. Complete the following exercises from [Section 16.7](#):

1(a)–(g), 7, 13(a)–(c)

Exercise 2. Let $\varphi: F_1 \rightarrow F_2$ be a homomorphism between fields. Prove that φ is injective.

Exercise 3. Let D be an integral domain of characteristic $p \neq 0$. Define $\varphi: D \rightarrow D$ by $\varphi(x) = x^p$.

- (a) Prove that φ is a homomorphism.
- (b) If D is finite, prove that φ is an isomorphism.

Exercise 4. The goal here is to explore the field of order 9.

- (a) Find an irreducible quadratic polynomial p in $\mathbb{Z}_3[x]$.
- (b) Then, $\mathbb{F}_9 = \{a + b\beta : a, b \in \mathbb{Z}_3 \text{ and } p(\beta) = 0\}$ is a field of order 9. Find the inverses of $1 + \beta$, $2 + \beta$, and $1 + 2\beta$.