Homework 8

MATH 231

Due Wednesday, October 25, 2023

Exercise 1. Complete the following exercises from Section 2.9 in the course textbook:

\# 1, 3, 5, 7, 9–16, 27–34

Exercise 2. Think through, but do not submit, answers/solutions to the following exercises from Section 2.9 in the course textbook:

\# 18, 19, 21, 22, 23, 24, 26

On Sunday, I took my kids to the park, and I got to talking to an economics professor whose kids were also at the park. He started complaining about how python's linear algebra library doesn't recognize column vectors and only uses row vectors. This caused particular confusion around computing the *outer product* of two vectors. I told him I didn't know what the outer product represented, and he explained its use in statistics for computing standard deviations and doing linear regressions. At some point, he claimed that the outer product is a projection matrix, and of course, I asked him to prove it. So, we did the following quick exercise together:

Exercise 3. Let $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$. Then the *outer product* of $\mathbf{v}$ and $\mathbf{w}$, denoted $\mathbf{v} \otimes \mathbf{w}$, is the $n \times n$ matrix defined by $\mathbf{v} \otimes \mathbf{w} = \mathbf{v}\mathbf{w}^T$. Recall the *dot product* (also known as the *inner product*) of $\mathbf{v}$ and $\mathbf{w}$, denoted $\mathbf{v} \cdot \mathbf{w}$, is defined by $\mathbf{v} \cdot \mathbf{w} = \mathbf{v}^T\mathbf{w}$.

Show that if $\mathbf{v} \cdot \mathbf{w} = 1$, then $(\mathbf{v} \otimes \mathbf{w})^2 = \mathbf{v} \otimes \mathbf{w}$. (This says that $\mathbf{v} \otimes \mathbf{w}$ is a projection matrix.)