Homework 12

MATH 231

Due Wednesday, December 6, 2023

Instructions. *Your work will be collected in class on the due date. We will also have a quiz in class on the due date based on the content from the assignment.*

Exercise 1. Complete the following exercises from Section 6.2 in the course textbook:

11, 13, 17, 19, 21, 35, 37, 38, 41

Exercise 2. Let $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and let $W = \text{span}\{\mathbf{v}\}$. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T(\mathbf{u}) = \text{proj}_W(\mathbf{u})$$

(In #41 in Section 6.2, you established that T is a linear transformation.) Find the matrix A satisfying $T(\mathbf{u}) = A\mathbf{u}$ for every $\mathbf{u} \in \mathbb{R}^2$.

Exercise 3. Complete the following exercises from Section 6.3 in the course textbook:

1, 3, 5, 7, 32

Exercise 4. Let A be an $n \times n$ orthogonal matrix. Show that if $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, then

$$(A\mathbf{u}) \cdot (A\mathbf{v}) = \mathbf{u} \cdot \mathbf{v}$$

Exercise 5. Let $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Show that R_θ is an orthogonal matrix.

Exercise 6. Let W be a subspace of $\mathbb{R}^n$. Show that $W = (W^\perp)^\perp$ (Hint: Use the orthogonal projection theorem.)

Exercise 7. Complete the following exercises from Section 6.4 in the course textbook:

1, 7, 13, 24, 26

Exercise 8. Complete the following exercises from Section 6.5 in the course textbook:

9, 15, 27, 33

Exercise 9. Complete the following exercises from Section 6.6 in the course textbook:

1, 5