1. CALCULUS

(1) Question 1.

(a) Calculate $\frac{d}{dx}\left(cx^c + c^2x^{-c}\right)$, where c is a non-zero constant.

(b) Determine y' where $y = \frac{1}{\sqrt{2-3x}}$.

(c) Let $f(r) = (r^3+1)(-5-r^4)$. Calculate $f'(r)$.

(2) This problem deals with the following function:

$$f(x) = \frac{4(x+1)(x-2)}{(x-2)(x-3)^2}$$

- Calculate all right-hand and left-hand limits of $f(x)$ at its vertical asymptotes.
- Does this function have a horizontal asymptote? Verify and explain.

(3) Given $\epsilon = 0.1$, determine $\delta > 0$ such that

$$|G(x) - 7| < \epsilon \text{ whenever } 0 < |x-3| < \delta.$$

2. COMBINATORICS

Let n be a positive integer. A **lecture hall partition** of length n is a partition $\lambda = (\lambda_n, \ldots, \lambda_2, \lambda_1)$ (where one or more λ_i may be zero) such that

$$0 \le \frac{\lambda_1}{1} \le \frac{\lambda_2}{2} \le \cdots \le \frac{\lambda_n}{n}.$$

Given $w = (w_1, w_2, \ldots, w_n) \in \tilde{C}_n/C_n$, create the partition $\lambda = (\lambda_n, \ldots, \lambda_2, \lambda_1)$ with

$$\lambda_j = \sum_{i=1}^{j} I_{i,j}.$$

This construction is a bijection between minimal length coset representatives of $\tilde{C}_n/C_n$ and lecture hall partitions of length n.

The runners corresponding to $i = 1$, 2, and 3 are runners 6, 5, and 3; $\lambda_{R(1)} = \lambda_1 = 12$, $\lambda_{R(2)} = \lambda_2 = 12$, $\lambda_{R(3)} = \lambda_5 = 7$, $\lambda_{r(1)} = \lambda_6 = 5$, $\lambda_{r(2)} = \lambda_7 = 5$, $\lambda_{r(3)} = \lambda_5 = 7$, and $\sigma_\lambda = (3,1,1)$. Therefore, $l\big(W(\sigma_\lambda)\big) = 3 + (12-5) + (12-5) + (7-7) + 0 \cdot 4 = 17$.