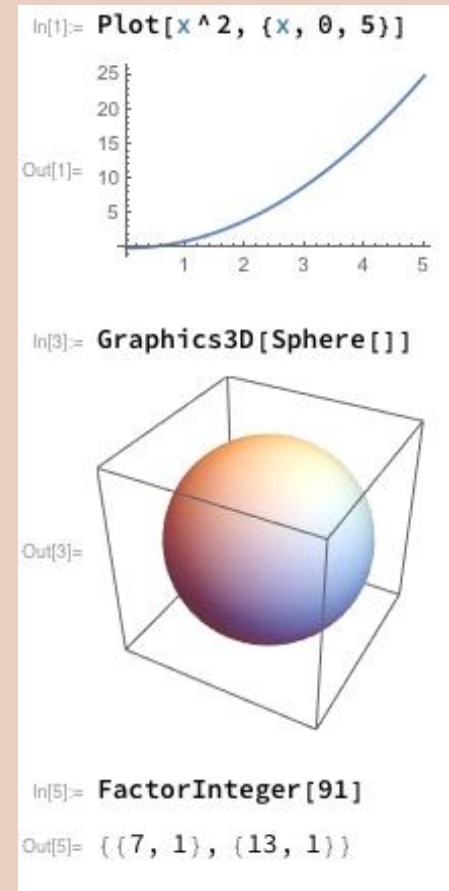


The Power and Pitfalls of *Mathematica* for 3D Design

**Christopher R. H. Hanusa
Queens College, CUNY**

My Mathematica Experience

- **Symbolic computational software**
- **Starting in 2008**
 - **Friendly syntax**
 - **Extensive documentation**
 - **Visualization capabilities**

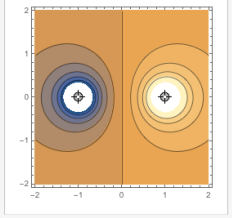


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Manipulate the electrostatic potential built from point charges:

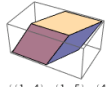
```
In[1]:= Manipulate[ContourPlot[q1 / Norm[{x, y} - p[[1]]] + q2 / Norm[{x, y} - p[[2]]], {{q1, -1}, -3, 3}, {{q2, 1}, -3, 3}, {{p, {{-1, 0}, {1, 0}}}, {-1, -1}, {1, 1}]]
```



Out[1]=

Create a simple polyhedron property explorer:

```
In[1]:= Manipulate[Column[{PolyhedronData[g], PolyhedronData[g, p]}, {g, PolyhedronData[All]}, {p, Complement[PolyhedronData /@ {"F", "E", "V"}]}], {g, AcuteGoldenRhombohedron}, {p, AdjacentFacelIndices}]
```



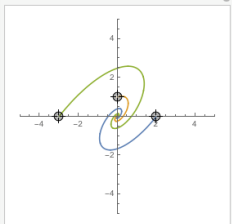
Out[1]=

```
{(1, 4), (1, 5), (4, 5), (2, 3), (3, 5), (2, 5), (1, 6), (4, 6), (3, 6)}
```

Visualize solutions to a linear system of differential equations $x' = Ax$:

```
In[1]:= A =  $\begin{pmatrix} -1.1 & 0.9 \\ -1.4 & 0.3 \end{pmatrix}$ ;
```

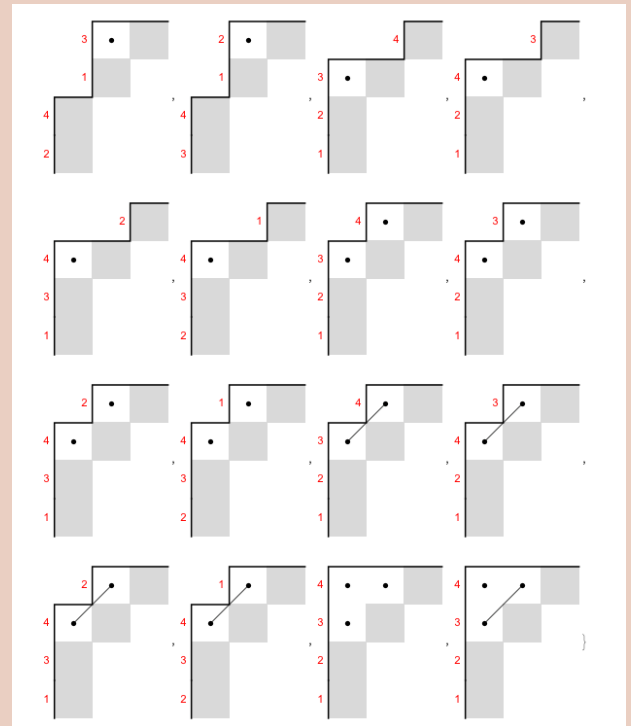
```
In[2]:= Manipulate[ParametricPlot[Evaluate[MatrixExp[A t, #] & /@ pt], {t, 0, 10}], {{pt, {{2, 0}, {0, 1}, {-3, 0}}}, Locator], SaveDefinitions -> True]
```



Out[2]=

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 - **Experimental Math**

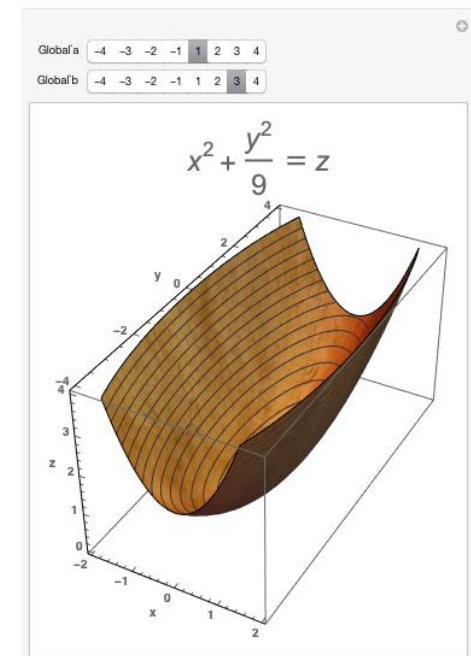


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 - **Experimental Math**
- **In my Teaching**
 - **Math Models, MV Calc, Math w/Mathematica**

Paraboloids

Investigate the behavior of quadric surfaces of the form $\pm \frac{x^2}{a^2} \pm \frac{y^2}{b^2} = z$



Teaching Methods

- Give students the tools to succeed
 - How to use the Documentation Center
 - Stand-alone tutorials
 - One-on-one help

Introduction to Lists

Math 213 - Math with Mathematica
Christopher Hanusa

Aim

In Mathematica, the key data structure is the list. Whenever multiple numbers are to be grouped together into a list, the entries of the list are separated by commas.

The aim of this tutorial is to introduce the user to lists, highlight important commands which generate lists, and to

These tutorials are meant to be interactive. You should be playing around with the inputs to try to see what ch

Throughout this and future tutorials, it is important to pay attention to the **syntax** of the commands. What inputs will know that you have mastered the command if you can create working Mathematica code involving the com

The `Range` command

We first start by creating simple lists of integers using the `Range` command.

A `Range` command has between 1 and 3 inputs; more inputs allow for more complex behavior. Compare the fo

```
Range[10]
```

```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

```
Range[2, 10]
```

```
{2, 3, 4, 5, 6, 7, 8, 9, 10}
```

```
Range[0, 10, 2]
```

```
{0, 2, 4, 6, 8, 10}
```

When there is only one input n , the output will be a list of integers starting at 1 and increasing to n .

When there are two inputs m and n , the output will be the list of integers starting at m and increasing to n .

When there are three inputs m , n , and `incr`, then the output will be the list of integers starting at m and increas

Comprehension Questions:

1. What do you think will happen if the input to `Range` is a negative integer? A non-integer? (To write a sentence, create a **new text cell** by clicking below this cell when the cursor

2. For each of the following `Range` commands, complete the following sub-questions.

(a) **BEFORE EVALUATING THE COMMAND**, what list do you expect the command to produce?

(b) Now, evaluate the command. Did it do what you expect it to do?

(c) If not, figure out what went wrong with your reasoning.

```
Range[1]
```

```
Range[Pi]
```

```
Range[10, 5]
```

```
Range[3, 4, 1/5]
```

```
Range[10, 30, Pi]
```

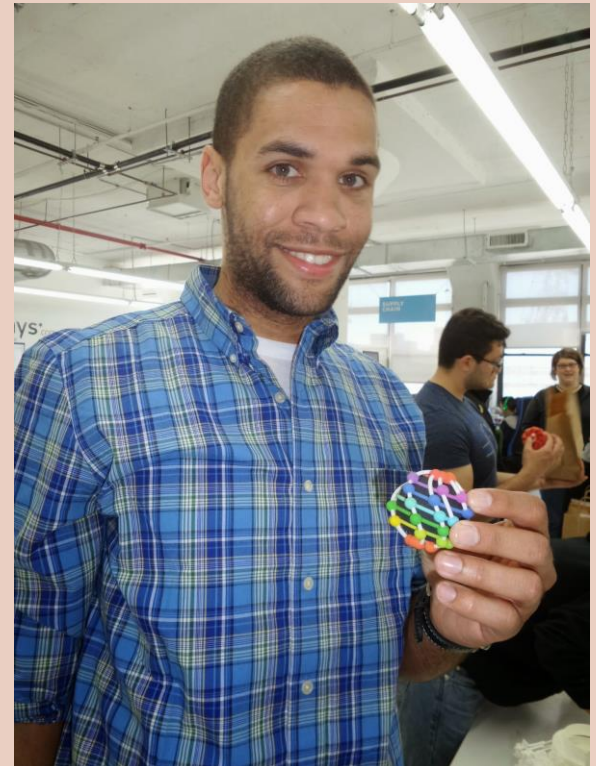
```
Range[Pi, 30, 10]
```

```
Range[100, 0, -8]
```

3. Determine which `Range` commands give the following lists.

Teaching Methods

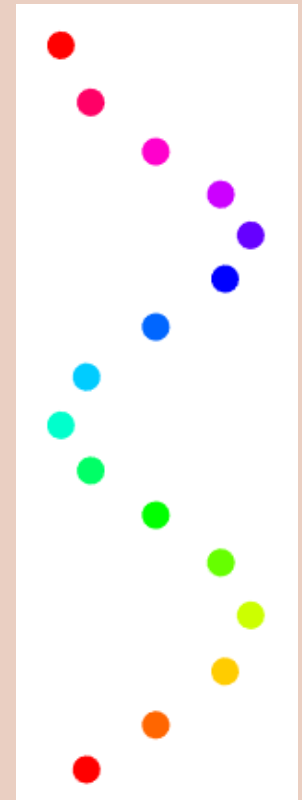
- **Give students the tools to succeed**
 - How to use the Documentation Center
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 - One-on-one help
- **Project-based learning**
 - Let them explore individually
 - Clear instructions and rubric
 - 3D printing since Spring '15



Mathematics in 3D Printing

Design in *Mathematica* requires **specifying coordinates**

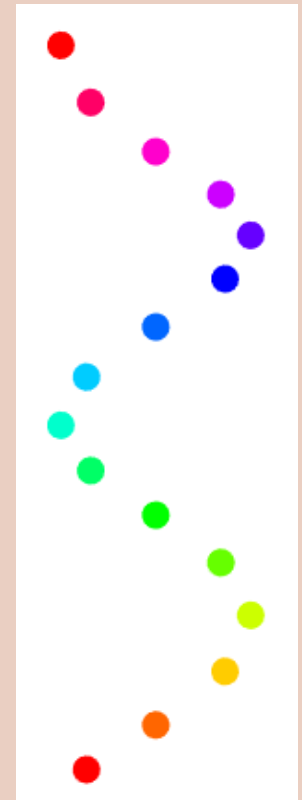
- 3D Coordinate System
- Transformations
 - Rotation, Translation, Scaling
- Parametric Curves / Vector Functions



Mathematics in 3D Printing

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- **3D Coordinate System**
- **Transformations**
 - Rotation, Translation, Scaling
- **Parametric Curves / Vector Functions**
- **Trigonometry**
 - Angles between lines, sine, arctan,
- **Geometric Objects**
 - Equations of Torus, Ellipsoid, ...
 - Polyhedra



Example: Geometry Basics

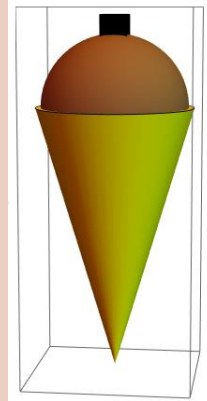
How does it work?

- **Coordinates for Primitives:**

`Sphere[{0, 0, 0}, .28]`

`Cuboid[{-.05, -.05, .26}, {.05, .05, .35}]`

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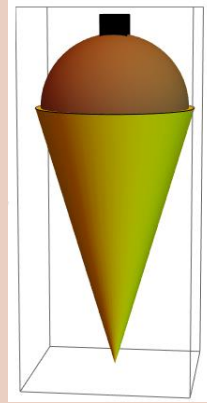
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What doesn't work?

- **Certain primitives not exportable!**
 - e.g. Tetrahedron, Pyramid



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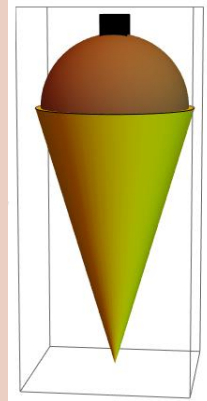
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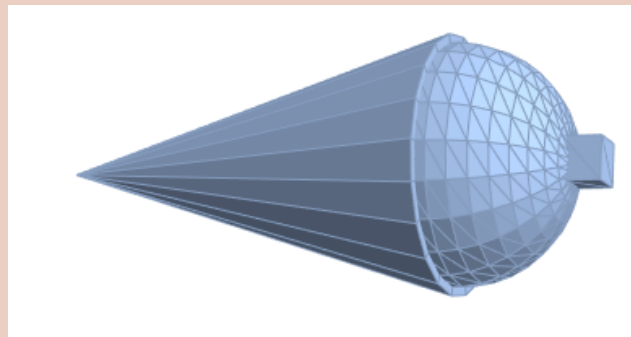
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- **Resolution in STL file is poor**

- Sphere, Cone
- Use `ParametricPlot3D`



Example: Curves and Surfaces

How does it work?

- BSplineCurve, Interpolation
- ParametricPlot3D
 - $f: \mathbb{R} \rightarrow \mathbb{R}^3$ plots a curve
 - $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ plots a surface

`ParametricPlot3D[... ,PlotStyle → Tube[.1]]`



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- **Secret option:** Extrusion



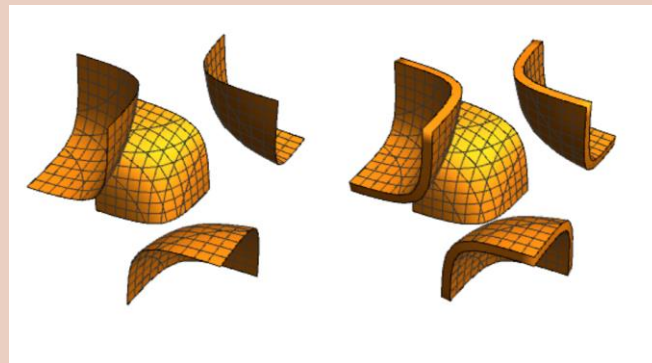
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- **Secret option:** Extrusion
 - Plot3D[$x^2 + y^2$, ... , Extrusion \rightarrow 1]
 - ContourPlot3D[... , Extrusion \rightarrow 0.1]



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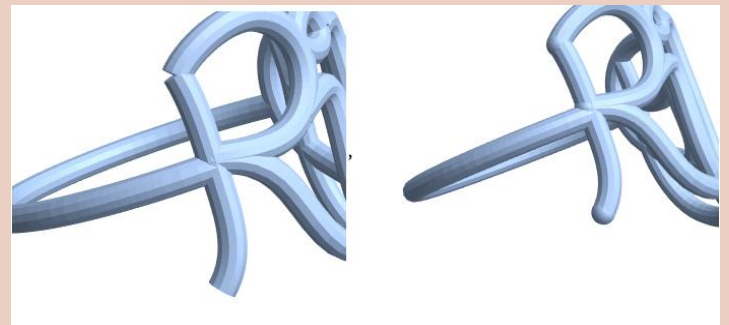
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- Secret option: Extrusion

What doesn't work?

- Can't export Tube[curve] .
 - Must add Tube in PlotStyle
- Caps of tubes are missing
 - Insert them independently and mind their resolution



Example: Polyhedra

How does it work?

- **PolyhedronData**
 - Imports curated polyhedra
 - Extract for modifications
 - Vertices, Incidences...



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 - Create your own polyhedron!



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What doesn't work?

- Time intensive to specify **EVERYTHING**
Is it manifold?



Example: MeshRegion Objects

How does it work?

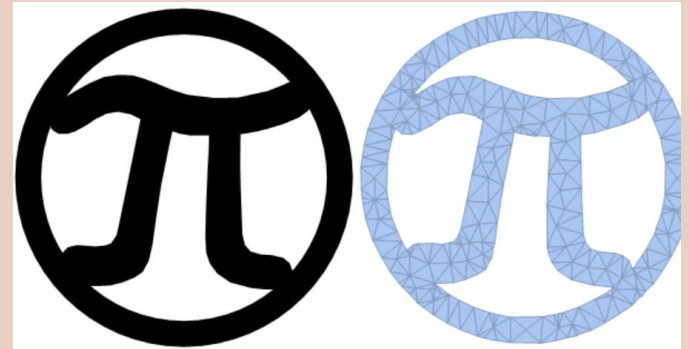
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 - **Triangulates 2D and 3D objects**



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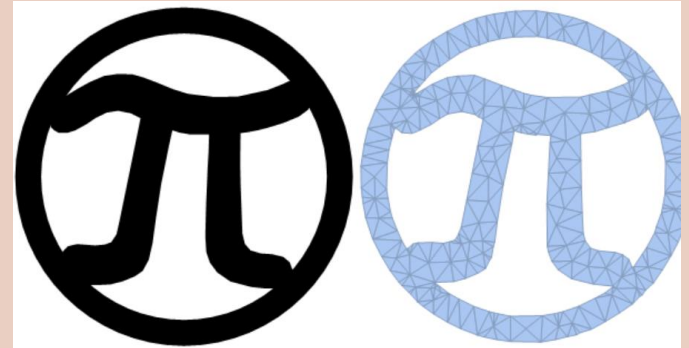
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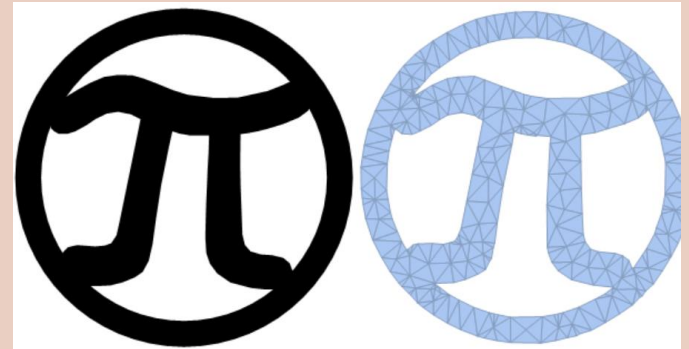
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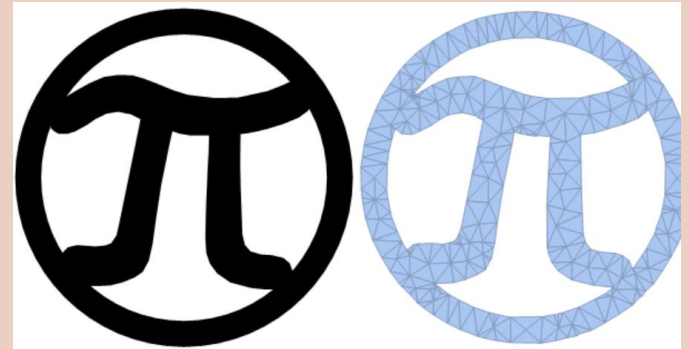
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What doesn't work?

- Can't intersect 3D MeshRegions!

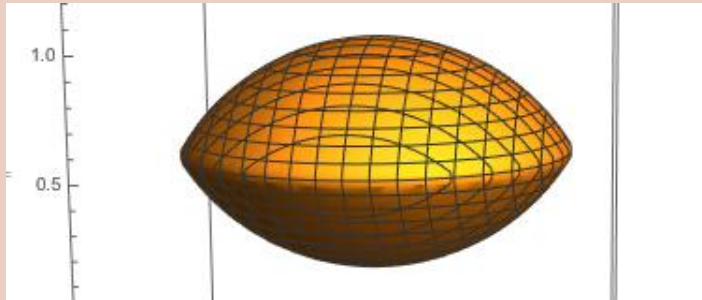


Example: RegionPlot3D

How does it work?

- Specify Boolean operations

$$x^2 + y^2 + z^2 \leq 1 \ \&\& \ x^2 + y^2 + (z - 1)^2 \leq 1$$

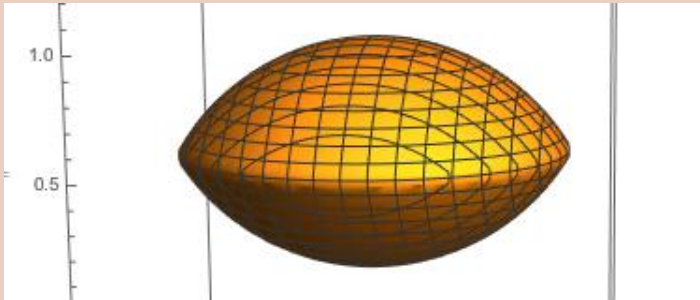


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- **Thicken a 2D surface**

```
PlotStyle -> {Directive[Thickness[.1]]}
```

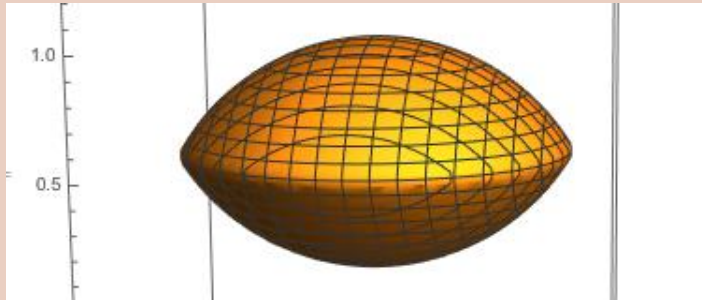


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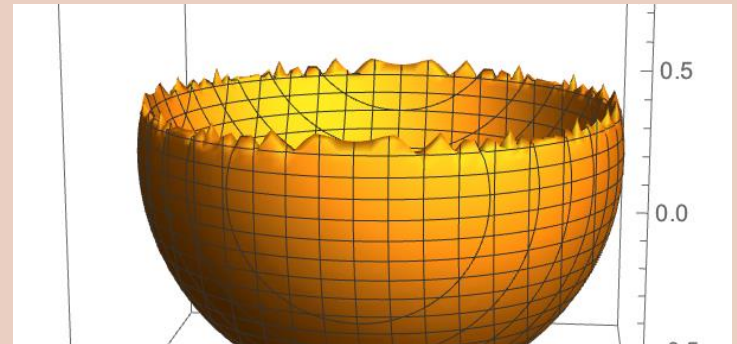


- Thicken a 2D surface

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PlotStyle -> {Directive[Thickness[.1]]}
```

What doesn't work?

- Poor edges / smoothing
 - Improve via PlotPoints



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 - **Notebook interface: No point and click!**
 - **Table** and **Map** to apply systematically

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Drawbacks

- 3D printing rather new to *Mathematica*.
- It doesn't "just work".
 - Certain commands not exportable at all
 - Never know when it will export or crash
 - Difficult to understand error messages

```
... BoundaryMeshRegion: The boundary curves self-intersect or cross each other in  
BoundaryMeshRegion[{{68.6021, 41.1552, 99.}, {85.7527, 51.444, 99.}, {85.7527, 51.  
75.4392, 152.}, {-33.282, 94.2991, 152.}, {-33.282, 94.2991, 99.}, {32.0944, 94.  
1>>]].  
... Join: Heads List and MeshCoordinates at positions 1 and 2 are expected to be the s  
... First: Nonatomic expression expected at position 1 in First[Fail].  
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- Colleagues use Sage

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- ***blog.wolfram.com***

THANK YOU!

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> Research > Talks

**Queens College
Students and Colleagues**



mathartshop.com

**Shapeways &
Lauren Slowik**

