How to succeed in your MατH class

Outline

- Getting the most out of lecture and discussion
- Getting the most out of your text and studying
- Test-taking strategies

Attending a

MατH

Class

How does a professor approach a class?

- Goal 1: Discuss key concepts (vocab, theorems)
- Goal 2: Discuss key mechanics (calculations, examples)
- Goal 3: Try to put course material in perspective.

Taking advantage of class

0. Get some sleep.
1. Prepare beforehand
2. GO TO CLASS
3. Listen
4. Take notes
5. Ask questions
6. Review your notes

Listening

- Listen actively
- Follow problem steps
- Anticipate where we're going
- Try to put the information in context
- If unclear, mark down key words, ask questions in class or later

Note taking

- Use copious amounts of paper.
- Leave lots of space.
- Write down what is written.
- Write down what is said
  - especially when it is a recap.
- Use shorthand if needed

- Revisit your notes later.
- Digest---rewrite in your own words

Common misconception

- "Material is taught in class."
  - preread; use class time for clarification
  - material is organized; prioritized
  - Much learning is done **outside** of class

Reading and Studying from a MατH Textbook

Words describing Math Textbooks

scary

condensed

difficult

incomprehensible

Example

DELTA NOTATION. Let f be a function. As usual, we let x stand for any argument of f, and we let y be the corresponding value of f. Thus, $y = f(x)$. Consider any number x_0 in the domain of f. Let Δx (read "delta x") represent a small change in the value of x, from x_0 to $x_0 + \Delta x$, and then let Δy (read "delta y") denote the corresponding change in the value of y. So, $\Delta y = f(x_0 + \Delta x) - f(x_0)$. Then the ratio

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

is called the *average rate of change* of the function f on the interval between x_0 and $x_0 + \Delta x$.

EXAMPLE 1: Let $y = f(x) = x^2 + 2x$. Starting at $x_0 = 1$, change x to 1.5. Then $\Delta x = 0.5$. The corresponding change in y is $\Delta y = f(1.5) - f(1) = 5.25 - 3 = 2.25$. Hence, the average rate of change of y on the interval between $x = 1$ and $x = 1.5$ is $\frac{\Delta y}{\Delta x} = \frac{2.25}{0.5} = 4.5$.

THE DERIVATIVE. If $y = f(x)$ and x_0 is in the domain of f, then by the *instantaneous rate of change* of f at x_0 we mean the limit of the average rate of change between x_0 and $x_0 + \Delta x$ as Δx approaches 0:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

provided that this limit exists. This limit is also called the *derivative* of f at x_0.

Start reading

Looks like class

No big deal

Example

NOTATION FOR DERIVATIVES. Let us consider the derivative of f at an arbitrary point x in its domain:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

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The value of the derivative is a function of x, and will be denoted by any of the following expressions:

$$D_x y = \frac{dy}{dx} = y' = f'(x) = \frac{d}{dx}\, y = \frac{d}{dx}\, f(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

The value $f'(a)$ of the derivative of f at a particular point a is sometimes denoted by $\left.\dfrac{dy}{dx}\right|_{x=a}$.

DIFFERENTIABILITY. A function is said to be *differentiable* at a point x_0 if the derivative of the function exists at that point. Problem 2 of Chapter 8 shows that differentiability implies continuity. That the converse is false is shown in Problem 11.

man I hate limits.

equals
equals
equals
equals

long word! should I be paying attention?

Example

Solved Problems

1. Given $y = f(x) = x^2 + 5x - 8$, find Δy and $\Delta y/\Delta x$ as x changes (*a*) from $x_0 = 1$ to $x_1 = x_0 + \Delta x = 1.2$ and (*b*) from $x_0 = 1$ to $x_1 = 0.8$.

(*a*) $\Delta x = x_1 - x_0 = 1.2 - 1 = 0.2$ and $\Delta y = f(x_0 + \Delta x) - f(x_0) = f(1.2) - f(1) = -0.56 - (-2) = 1.44$.

So $\dfrac{\Delta y}{\Delta x} = \dfrac{1.44}{0.2} = 7.2$

(*b*) $\Delta x = 0.8 - 1 = -0.2$ and $\Delta y = f(0.8) - f(1) = -3.36 - (-2) = -1.36$. So $\dfrac{\Delta y}{\Delta x} = \dfrac{-1.36}{-0.2} = 6.8$

Geometrically, $\Delta y/\Delta x$ in (*a*) is the slope of the secant line joining the points $(1, -2)$ and $(1.2, -0.56)$ of the parabola $y = x^2 + 5x - 8$, and in (*b*) is the slope of the secant line joining the points $(0.8, -3.36)$ and $(1, -2)$ of the same parabola.

2. If a body (that is, a material object) starts out at rest and then falls a distance of s feet in t seconds, then physical laws imply that $s = 16t^2$. Find $\Delta s/\Delta t$ as t changes from t_0 to $t_0 + \Delta t$. Use the result to find $\Delta s/\Delta t$ as t changes: (*a*) from 3 to 3.5, (*b*) from 3 to 3.2, and (*c*) from 3 to 3.1.

$$\frac{\Delta s}{\Delta t} = \frac{16(t_0 + \Delta t)^2 - 16t_0^2}{\Delta t} = \frac{32t_0\Delta t + 16(\Delta t)^2}{\Delta t} = 32t_0 + 16\,\Delta t$$

(*a*) Here $t_0 = 3$, $\Delta t = 0.5$, and $\Delta s/\Delta t = 32(3) + 16(0.5) = 104$ ft/sec.

(*b*) Here $t_0 = 3$, $\Delta t = 0.2$, and $\Delta s/\Delta t = 32(3) + 16(0.2) = 99.2$ ft/sec.

(*c*) Here $t_0 = 3$, $\Delta t = 0.1$, and $\Delta s/\Delta t = 97.6$ ft/sec.

Since Δs is the displacement of the body from time $t = t_0$ to $t = t_0 + \Delta t$,

$$\frac{\Delta s}{\Delta t} = \frac{\text{displacement}}{\text{time}} = \text{average velocity of the body over the time interval}$$

- Some problems.
- Looks like I could do them.
- Yup, those answers look correct,
- skim... skim...

Example

Supplementary Problems

14. Find Δy and $\Delta y/\Delta x$, given
(*a*) $y = 2x - 3$ and x changes from 3.3 to 3.5.
(*b*) $y = x^2 + 4x$ and x changes from 0.7 to 0.85.
(*c*) $y = 2/x$ and x changes from 0.75 to 0.5.

Ans. (*a*) 0.4 and 2; (*b*) 0.8325 and 5.55; (*c*) $\frac{4}{3}$ and $-\frac{16}{3}$

15. Find Δy, given $y = x^2 - 3x + 5$, $x = 5$, and $\Delta x = -0.01$. What then is the value of y when $x = 4.99$?

Ans. $\Delta y = -0.0699$; $y = 14.9301$

16. Find the average velocity (see Problem 2), given: (*a*) $s = (3t^2 + 5)$ feet and t changes from 2 to 3 seconds. (*b*) $s = (2t^2 + 5t - 3)$ feet and t changes from 2 to 5 seconds.

Ans. (*a*) 15 ft/sec; (*b*) 19 ft/sec

17. Find the increase in the volume of a spherical balloon when its radius is increased: (*a*) from r to $r + \Delta r$ inches; (*b*) from 2 to 3 inches. (Recall that volume $V = \frac{4}{3}\pi r^3$.)

Ans. (*a*) $\frac{4}{3}\pi[3r^2 + 3r\,\Delta r + (\Delta r)^2]\,\Delta r$ in^3; (*b*) $\frac{76}{3}\pi$ in^3

Homework.
Tomorrow!
What were the problems again?
Oh, look at my email & that fun webpage.
Right, hwk.

Example

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Ummm.....

Delta y over Delta x
I have an equation
x changing

I don't understand

Maybe I should skip this one...

What happened?

- Reading for speed.
- Not focused
  - background activities
  - not prepared => distractions.

What to do?

- Read for comprehension.
  - Different speeds.
    - General idea: skim
      - Circle unfamiliar words.
    - Detail: read slowly and reread.
      - Mathematicians hate fluff.

What to do?

- Read for comprehension.
  - Multiple times!
    - Skim once before class
    - Difficult sentences: 2-5 times
    - Even, read aloud.

What to do?

- Simple examples measure comprehension
  - Read example SLOWLY
  - Fill in missing steps.
  - Mark in margin concept learned.

Math books are not novels

- For best results:
  - Read in quiet surroundings.
  - Have a designated “reading time”.
  - Have everything ready before starting.
- Make notes in your book.
  - More information is better.
  - Post-its? Highlighters?

Highlighters gone wild!

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R A I N B O W

Notes serve purposes.

- Itch to highlight? Write either:
- What it does.
  - Introduces an important idea.
  - Explains how to apply important idea.
  - Important technique to remember
- What it says.
  - Definition of $\Delta y / \Delta x$
  - Example using $\Delta y / \Delta x$
- Something that doesn't make sense!

- Now, let's redo the example
  - correctly

Example

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Read slowly!

If it looks like class and is confusing, Look at your notes.

TIPS

Where: No distractions
With all the tools you need.
Alone/In groups/with tutor/in math help room

Reading: when something is not clear,
reread it slowly
parse it step by step.
class notes may help.

Writing: keep a study notebook nearby
keep track of vocabulary
keep track of theorems

TIPS

Worked Problems:
Cover up solutions and do on own.
Write down important concepts/approaches.
Reality checks.

Homework:
DO IT!
Cover up answers and try multiple approaches.
Do NOT work backwards.
Realize that there is a mathematical **approach**.
Approach from a test POV

Test-taking Skills

Pre-Test

Preparation:
Work continually during the weeks beforehand
On test week: review notes and exercises
Redo exercises.
Make a cheat sheet.

Create a practice test of your own,
with one or two problems from each section
Take in a test-like environment

Night before: GET SLEEP!!!

At the test

Read over the entire exam.
First do the problems you know you can do.
Prioritize the rest.

Read each question carefully
ANSWER THE QUESTION!
Do reality checks.

Don't erase; cross out.
Show all your work

If you have time, redo some problems.

References I:

- Engaging Ideas: The Professor's Guide to Integrating Writing, Critical Thinking, and Active Learning in the Classroom
- Chapter 8: Helping Students Read Difficult Texts

- Two websites on how to read a math textbook:

- http://wc.pima.edu/~carem/Mathtext.html
- http://academic.cuesta.edu/acasupp/AS/704.htm
- http://salsa.missioncollege.org/mss/stories/storyReader$4

References II:

- Study methods:

- http://wc.pima.edu/~carem/Mathtext.html
- http://salsa.missioncollege.org/mss/stories/storyReader$41
- http://euler.slu.edu/Dept/SuccessinMath.html